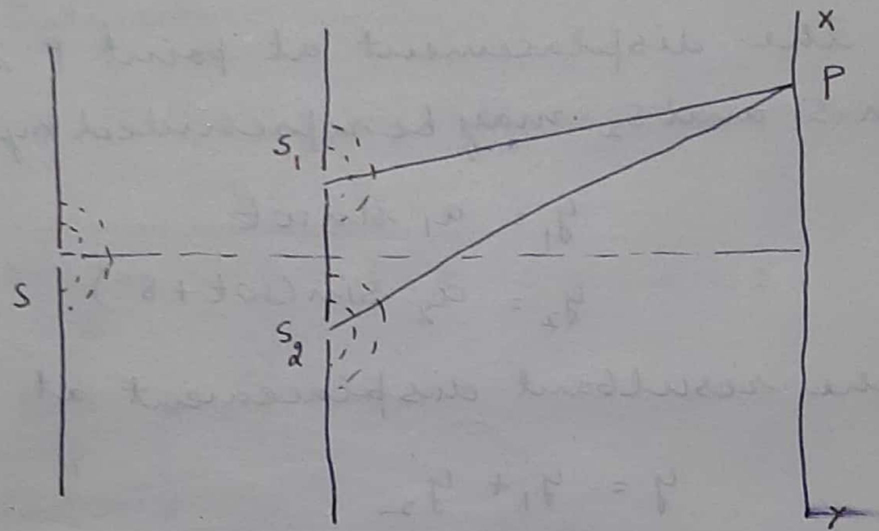
Young's Double Slit Experiment:

Thomas Young, in 1801 demonstrated the phenomenon of interference of light. It comes under the category of interference due to division of wave front.

His arrangement is as follows



Let S be a narrow slit illuminated by a monochromatic light source. S_1 and S_2 are two parallel slits very close together and equidistant from the 'S.' The waves from S that reach S_1 & S_2 are in same phase. Then, beyond S_1 and S_2 they travel as if they started from S_1 and S_2. A screen XY is placed in front of S_1 and S_2 where the interference pattern is formed.

Expression for maximum and minimum Intensity:

Let a_1 and a_2 be the amplitudes of the wave from S_1 and S_2 respectively.

The waves arrive at P having traversed diff paths S_1P and S_2P. Hend the phase difference at P is

$$\delta = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda}(S_2P - S_1P)$$

where λ: wavelength of light.

Then the displacement at point P due to waves from S_1 and S_2 may be represented by

$$y_1 = a_1 \sin\omega t$$

$$y_2 = a_2 \sin(\omega t + \delta)$$

So the resultant displacement at P, is

$$y = y_1 + y_2$$

$$= a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$

$$= a_1 \sin\omega t + a_2 \sin\omega t \cos\delta + a_2 \cos\omega t \sin\delta$$

$$= \sin\omega t(a_1 + a_2\cos\delta) + \cos\omega t(a_2 \sin\delta)$$

Let

$$\left.\begin{aligned} a_1 + a_2\cos\delta &= R\cos\theta \\ a_2\sin\delta &= R\sin\theta \end{aligned}\right\} \begin{aligned} &\to (1) \\ &\to (2) \end{aligned}$$

$$\therefore y = \sin\omega t\, R\cos\theta + R\cos\omega t \sin\theta$$

$$= R\sin(\omega t + \theta)$$

Hence resultant displacement at P is simple harmonic of amplitude R.

where R is (

$$\boxed{R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta}$$

The resultant intensity at P is proportional to R^2

$$I \propto R^2$$

for simplicity let the constant of proportionality = 1

$$\therefore I = R^2$$

$$\boxed{I = a_1^2 + a_2^2 + 2a_1a_2 \cos\delta} \longrightarrow (3)$$

Maximum Intensity:

Intensity will be maximum when

$$\cos\delta = 1$$

condition for maxima

phase diff^n $\rightarrow \delta = 2n\pi$ $\quad n = 0, 1, 2 \ldots$

or

path diff^n $(S_2P - S_1P)$ $\leftarrow$ P.D $= n\lambda$

$$\boxed{I_{max} = (a_1 + a_2)^2}$$

⇒ "The maximum intensity is greater than the sum of two sperate intensities."

Minimum Intensity:

Intensity will be minimum when

$$\cos\delta = -1$$

$$\therefore \delta = (2n+1)\pi \quad n = 1, 2, 3 \ldots$$

or

$$\text{Path diff} = S_2P - S_1P = (2n+1)\frac{\lambda}{2}$$

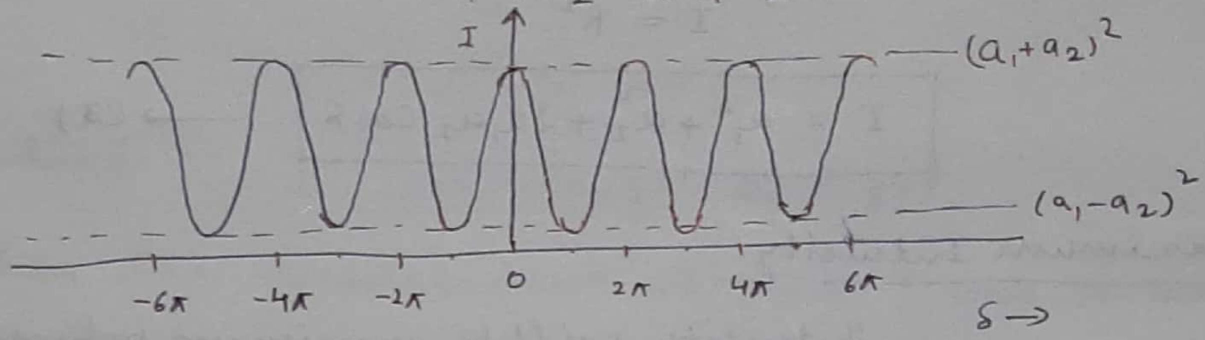
$$\boxed{I_{min} = (a_1 - a_2)^2}$$

" The mininmum intensity is less than diff of two seperate intensities."

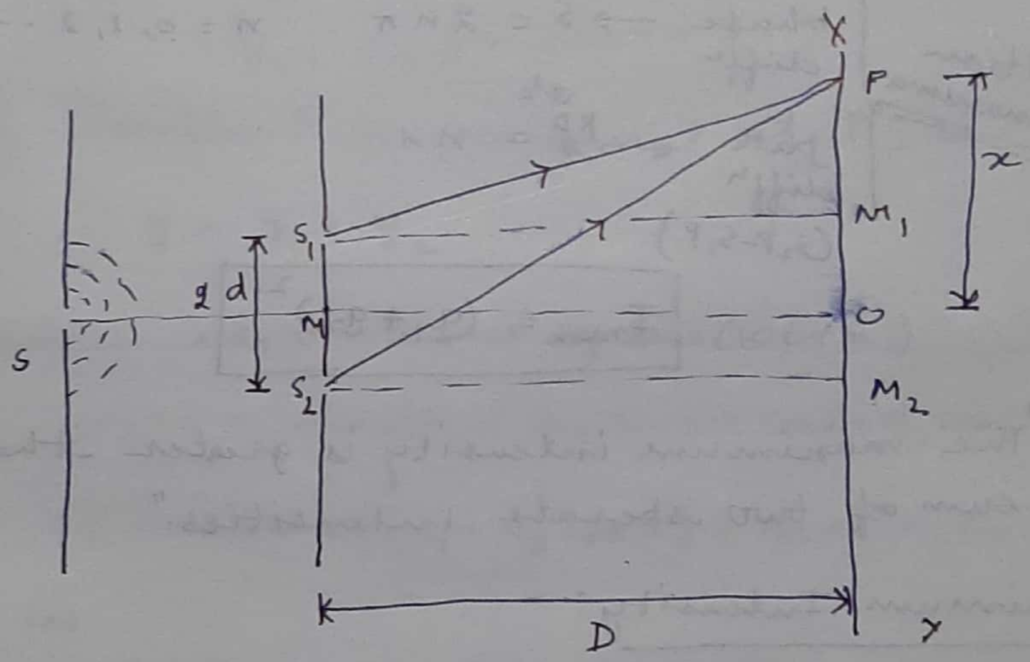
Intensity Distribution Curve:

The resultant intensity of light at any point on the screen is given by

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos\delta$$



Spacing between two consecutive maxima or Minima:



Let M be the point that bisect S_1S_2. From M draw a perpendicular MO onto the screen XY. Let P be any point on the screen.

Let

$S_1S_2 = 2d$ (spacing b/w slits)

$MO = D$ (spacing b/w slit & screen)

$OP = x$

Let us calculate the path difference $SP_2 - SP_1$.

$$(S_2P)^2 = (S_2M_2)^2 + (PM_2)^2$$ where S_2M_2 is perpendicular to screen.

$$= D^2 + (x+d)^2$$

$$= D^2\left[1 + \frac{(x+d)^2}{D^2}\right]$$

$$\therefore S_2P = D\left[1 + \frac{(x+d)^2}{D^2}\right]^{1/2}$$

Since $D >> x+d$

$$\therefore S_2P = D\left[1 + \frac{1}{2}\frac{(x+d)^2}{D^2}\right]$$ (using Binomial Theorem)

$$S_2P = D + \frac{1}{2}\frac{(x+d)^2}{D} \longrightarrow (1)$$

Similarly,

$$S_1P = D + \frac{1}{2}\frac{(x-d)^2}{D} \longrightarrow (2)$$

$$\therefore S_2P - S_1P = \frac{1}{2D}\left[(x+d)^2 - (x-d)^2\right]$$

$$S_2P - S_1P = \frac{2xd}{D} \longrightarrow (3)$$

For bright fringes (Maxima)

$$S_2P - S_1P = \frac{2xd}{D} = n\lambda$$ where $n = 0, 1, 2, \ldots$

$$x = \frac{D}{2d} n\lambda \longrightarrow (4)$$

For dark fringes (Minima)

$$S_2P - S_1P = \frac{2xd}{D} = (2n+1)\frac{\lambda}{2}$$ $n = 0, 1, 2 \ldots$

$$x = \frac{D}{2d}(2n+1)\lambda/2 \rightarrow (5)$$

Now let x_n and x_{n+1} denote distance b/w n^{th} and $(n+1)^{th}$ bright ~~p~~ fringes

$$\therefore \quad x_n = \frac{D}{2d} n\lambda$$

$$x_{n+1} = \frac{D}{2d}(n+1)\lambda$$

Spacing b/w consecutive bright fringes

$$\boxed{x_{n+1} - x_n = \frac{D}{2d}\lambda}$$

Similary spacing b/w consecutive dark fringes

$$x_{n+1} - x_n = \frac{D}{2d}\lambda$$

$\therefore$ $\frac{D}{2d}\lambda = W$ is called fringe width